arXiv:hep-ph/0103158v2 1 Jun 2001

Limitations of the Φ measure of fluctuations in event-by-event analysis

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February 1, 2008

Abstract

We provide a critical overview of the Φ measure of fluctuations and correlations. In particular we show that its discriminating power is rather limited in situations encountered in experiment.

PACS numbers: 25.75.-q 24.60.-k 05.20.-y 05.70.Ln

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It is widely recognized that event-by-event analysis of experimental data on multiparticle reactions (and especially studies of fluctuation patterns seen there) provides us with very important and sensitive tool in our attempts to understand dynamics of heavy ion collisions [1]. They are particularly useful in searching for some special features of the quark-gluon plasma (QGP) equation of state [2]. The question of the best method of their investigation allowing for the most information to be gathered is therefore of great interest (cf., for example, reviews [3] and references therein). Some time ago, a novel method of investigation of fluctuations in even-by-event analysis of high energy multiparticle production data was proposed and applied to nuclear collisions [4]. It is based on a suitably defined measure Φ,

$$\Phi_x = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\bar{z^2}} \qquad \text{where} \qquad Z = \sum_{i=1}^{N} z_i, \tag{1}$$

which originally was supposed to discriminate wheather or not fluctuations of a given observable x is exactly the same for nucleon-nucleon and nucleus-nucleus collisions [5]. Here $z_i = x_i - \bar{x}$ where $\bar{x}$ denotes the mean value of the observable x calculated for all particles from all events (the so called inclusive mean) and N is the number of particles analysed in the event. In (1) $\langle N \rangle$ and $\langle Z^2 \rangle$ are averages of event-by-event observables over all events, whereas the last term is the square root of the second moment of the inclusive z distribution. By construction $\Phi_x = 0$ for independently produced particles [4].

However, application of this method is not free from controversy. When first applied to NA49 data for central $Pb - Pb$ collisions at 158 A·GeV [6] it apparently revealed that fluctuations of transverse momentum ($x = p_T$) decreased significantly with respect to elementary NN collisions. This in turn has been interpreted as a possible sign of equilibration taking place in heavy ion collisions, providing thus an enviroment for the possible creation of QGP. It was immediately realised that existing models of multiparticle production are leading in that matter to conflicting statements [7]. The more recent NA49 data [9] reported, however, a new value (almost an order of magnitude greater then the previous one), which was the one corresponding to a pion gas in global equilibrium [8]. A number of attempts followed, trying to clarify the meaning of Φ (cf., for example, [10, 11] and references therein.) In the mean time, it was extended to study event-by-event fluctuations of "chemical" (particle type) composition of produced secondaries [12], to study azimuthal correlations among them (which are important for studies of flow patterns observed in heavy ion collisions [13]) and to cover also higher order correlations [14]. Finally, Φ has been also analysed by means of the nonextensive statistic both for p_T correlations [15] and for fluctuations of chemical composition as well [16].

In [16] we have pointed that, if there are some additional fluctuations (not arising from quantum statistics, like those caused by the experimental errors), which add in the same way to both terms in definition (1) of Φ, it would perhaps be better to use another form of it, like for example

$$\Phi \;\rightarrow\; \Phi^{\star} \;=\; \frac{\langle Z^2\rangle}{\langle N\rangle} - \bar{z^2}, \tag{2}$$

where the variances σ_x^2 would cancel (being present only implicitly). We would like to elaborate on this criticism towards Φ (as well as $\Phi^{\star}$) measure in more detail here. Our point is (cf. also [3]) that inclusive experiments provide us both with single particle distributions $P(x)$ (i.e., with information on fluctuations of the x-values) and with multiplicity distributions $P(N)$ (i.e., with information on fluctuations of N and, when put together with $P(x)$, also with information on correlations between x-value and multiplicity N). Also correlations between produced particles (i.e., between x-values), especially Bose-Einstein correlations (BEC) resulting from their statistics, are to a large extend known [17]. Measures Φ (or $\Phi^{\star}$) depend on all of them, because (notice that $\bar{z^2} = \sigma_z^2 = \sigma_x^2$)

$$\langle Z^2\rangle \;=\; \sigma_Z^2 \;=\; \langle N\rangle\sigma_x^2 \;+\; \langle N(N-1)\rangle \cdot \mathrm{cov}(x_i, x_j) \;+\; c(x, N), \tag{3}$$

where covariance $\mathrm{cov}(x_i, x_j) = \overline{x_i x_j} - \bar{x}_i\bar{x}_j = \rho\sigma_x^2$, i.e., is given in terms of the variance σ_x^2 of the variable x and their mutual correlation coefficient ρ, while the last term describes the possible correlation between the variable x and the multiplicity N. Notice that assuming, for example, $\langle x\rangle_N$ for given N being given by

$$\langle x\rangle_N \;=\; \bar{x}\left[1 \;+\; \alpha\cdot\frac{N - \langle N\rangle}{\langle N\rangle}\right] \tag{4}$$

one gets for correlation term

$$c(x, N) \;=\; \alpha^2\cdot\bar{x}^2\cdot\left(\langle N^2\rangle - \langle N\rangle^2\right). \tag{5}$$

Therefore $\Phi = 0$ is achieved only for independent x-values, in which case they are also uncorrelated. If this is not the case they are given by:

$$\begin{aligned}
\Phi &= \sqrt{\sigma_x^2 \;+\; \frac{\langle N(N-1)\rangle}{\langle N\rangle}\cdot\mathrm{cov}(x_i, x_j) \;+\; \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}\alpha^2\bar{x}^2} \;-\; \sqrt{\sigma_x^2} \\
&= -\sigma_x \;+\; \sqrt{\Phi^{\star} + \sigma_x^2}, \qquad (6)\\
\Phi^{\star} &= \left(\langle N\rangle - 1 + \frac{\sigma_N^2}{\langle N\rangle}\right)\cdot\mathrm{cov}(x_i, x_j) \;+\; \frac{\sigma_N^2}{\langle N\rangle}\alpha^2\bar{x}^2 \\
&= \Phi(\Phi \;+\; 2\sigma_x). \qquad (7)
\end{aligned}$$

Because in reality (neither in experiments nor in models or event generators attempted to describe them) conditions of independence are not fulfilled, $\Phi,\ \Phi^\star \neq 0$ and a question arises: what information do they convey? In particular, can we learn more from event-by-event analysis performing it by means of the Φ (or $\Phi^\star$) measure rather than by being satisfied with the known inclusive distributions alone? In this respect one should notice that [3]:

- Both Φ and $\Phi^\star$ depend strongly on the multiplicity fluctuations and this dependence vanishes only in the limiting case of a Poisson distribution (for which $\langle N^2\rangle - \langle N\rangle^2 = \langle N\rangle$ and $\langle N(N-1)\rangle = \langle N\rangle^2$) where, for example,

$$\Phi^\star \ = \ \langle N\rangle \cdot \mathrm{cov}(x_i, x_j) \ + \ \alpha^2 \bar{x}^2. \tag{8}$$

- Experimental data allow us to study the dependence of $\langle x\rangle_N$ on the multiplicity N in an explicit way. In the case when such correlations are absent, the $\Phi^\star$ measure reduces simply to a two-particle correlation measure

$$\Phi^\star \ = \ \left(\langle N\rangle - 1 \ + \ \frac{\sigma_N^2}{\langle N\rangle}\right) \cdot \mathrm{cov}(x_i, x_j). \tag{9}$$

- The influence of the two-particle correlation term $\mathrm{cov}(x_i, x_j)$ depends on the kind of particles involved: $\Phi = 0$ for Boltzman statistics, $\Phi < 0$ for fermions and $\Phi > 0$ for bosons. However, this type of correlations, especially for BEC for bosons, are subject to extensive experimental and theoretical investigations where one mesures or modells the so called $2-$particle correlation function $C_2(x_i, x_j)$ [17]. Because

$$\mathrm{cov}(x_i, x_j) \ = \ \int\int \, dx_i dx_j \, x_i \frac{dn}{dx_i} \, x_j \frac{dn}{dx_j} \, \left[C_2(x_i, x_j) \ - \ 1\right], \tag{10}$$

  it means that Φ and $\Phi^\star$ measures are nothing but the correlation measure C_2 averaged over single particle distributions dn/dx (i.e., we are in fact not gaining but losing some information contained in C_2 and dn/dx and none of Φ's provide us with any new information).

The above remarks, being obvious and essentially known, [3, 10], have so far not been backed by any convincing numerical illustration. By this we mean calculations using an event generation algorithm which would satisfy all conservation laws (especially energy-momentum conservation) and at the same time model also the BEC. Because both points seem to play a major role in a proper description of Φ (or $\Phi^\star$) measure [3, 9], which was not checked properly [21], we would like to fill this gap by calculating Φ and $\Phi^\star$ in simple models of hadronization using an algorithm which preserves

both energy-momentum conservation and the original single-particle distributions and models at the same time all features of BEC in such processes [18]. To this end we shall compare Φ and $\Phi^\star$ measures as function of mean multiplicity $\langle N^{(-)}\rangle$ of negatively charged particles for hadronization processes proceeding without and with BEC. Two simple models of hadronizations of mass M will be considered: the cascade model (CAS) developed by us recently [19] (where the whole space-time and phase-space history of the hadronization process is explicitly known) and simple statistical model (MaxEnt) based on information theory approach proposed in [20] (where details of hadronization are not available; both models were already used in [18]).

Most of the above mentioned applications of Φ measure concerned transverse momenta $x = p_T$ [4, 6, 8, 9], which is always positive. The corresponding variable here would be $|p|$ [22], results for which are shown in Fig. 1. We have found it interesting to enlarge analysis for the variable which can take any sign, as momentum p in our case, results for which are shown in Fig. 2. The differences are striking. Whereas behaviour of $\Phi(x = |p|)$ is similar to that for $x = p_T$, that of $\Phi(x = p)$ is much more sensitive to the limits imposed by the energy-momentum conservation (both measures are negative here). In both Figs. 1 and 2 we present Φ and $\Phi^\star$ for CAS and MaxEnt models without and with BEC, in this later case for two different choices of the weights specifying BEC [23]: constant $P = 0.5$ and Gaussian P as discussed in [18] (which lead to very different BEC patterns). The results are given as functions of the mean multiplicity $\langle N^{(-)}\rangle$ of negatively charged secondaries produced in hadronization process of mass M.

The special feature emerging from our calculation is the fact that the effect caused by BEC is clearly visible in all cases (it is maximal for the case of constant weights because the BEC is maximal there [18]). To make dependence on BEC more clear we present in Fig. 3 for $x = |p|$ case the corresponding differences $\delta\Phi = \Phi_{BEC} - \Phi_{noBEC}$ and $\delta\Phi^\star = \Phi^\star_{BEC} - \Phi^\star_{noBEC}$. The curves show the best power fits of the type $\delta\Phi \propto \langle N^{(-)}\rangle^\delta$ exhibiting very strong and growing dependence of the effect of BEC on $\langle N^{(-)}\rangle$. There are some other characteristic features of the results worth attention, namely: (i) the scale of effect is different in Φ and $\Phi^\star$ this reflects differences in their definitions (6) and (7); (ii) their different dependencies on $\langle N^{(-)}\rangle$ indicate that σ_x plays a significant role here; (iii) the completely different behaviours for $x = p$ and $x = |p|$ (compare Fig. 1 with Fig. 2) illustrate how Φ_x changes dramatically whether the measured variable is restricted to being positive or not. Because in the second case the role of the energy-momentum constraints is much more important, dependence on this constraint is more pronounced. Notice that points for MaxEnt in Figs. 1 and 2 are consistently higher

than for CAS. This should probably be attributed to different ways of imposing conservation laws in both models: they are satisfied globally in the statistical model MaxEnt and locally, in every branching point, in the cascade model CAS. Because of this CAS prefers symmetric distributions in momenta (here forward-backward) whereas MaxEnt allows more frequently for asymmetric ones (for example, when one particle with large momentum is balanced by a bunch of low momenta particles in other hemisphere). The difference between both types of models seen in Fig. 3 originate from different shapes of $C_2(p_i, p_j)$ functions, which are broader (what corresponds to smaller "radius" R) for CAS type of models.

To conclude: we have demonstrated using numerical algorithm which preserves energy-momentum conservation and at the same time models BEC in multiparticle hadronization that Φ (and also $\Phi^\star$) measure is very sensitive both to the constraints provided by conservation laws and to the effects of correlations (exemplified here by the BEC). Both features must therefore be carefully accounted for when attempts are made to reach some conclusions concerning new physical effects when using these measures to the event-by-event analysis of data. We have also written down explicit relations between both measures and all other observables (well defined and known in statistics) available from inclusive experiments. They show explicitly that both Φ's are closely related to inclusive correlation functions. In this respect we believe that no new information is obtained from these measures in comparison with what is available from inclusive measurements.

The partial support of Polish Committee for Scientific Research (grants 2P03B 011 18 and 621/E-78/ SPUB/ CERN/ P-03/ DZ4/99) is acknowledged.

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[22] Because so far our numerical algorithm for BEC is available only for essentially one dimensional hadronization with p_T-dependence summarized by transverse mass μ_T, the $x = |p|$ variable is the only choice we have for this kind of comparison.

[23] In [18] in addition to the constant weigths arbitrary chosen as $P = 0.5$, also a kind of the "most natural" choice of weights was considered (using only the information provided by event generator) namely, for a pair of particles (ij) weight $P(ij) = \exp\left[-\frac{1}{2}\delta^2_{ij}(x) \cdot \delta^2_{ij}(p)\right]$ for CAS (particles (ij) described by spacio-temporal wave packets separated by $\delta_{ij}(x)$ would have widths given by their momentum separation $\delta_{ij}(p)$) and weight $P(ij) = \exp\left[-\frac{\delta^2_{ij}(p)}{2\mu_T T_l}\right]$ for MaxEnt (μ_T is transverse mass put here to be equal 0.3 GeV and the role of spatial dimension is now played by the "temperature" T_l of the l^{th} event).

Figure Captions:

Fig. 1 Φ (in GeV, left panels) and $\Phi^\star$ (in GeV^2, right panels) as function of mean (negatively charged) multiplicity $\langle N^{(-)} \rangle$ calculated for $x = |p|$ for two different hadronization models, CAS and MaxEnt, without and with BEC. Upper panels contain BEC obtained using constant weights, lower panels contain BEC corresponding to Gaussian weights [23]. Black symbols denote CAS and MaxEnt models with BEC, open symbols without BEC. Stars correspond to MaxEnt and circles to CAS models, respectively. Lines are just interpolating between calculated points.

Fig. 2 The same as in Fig. 2 but this time for $x = p$.

Fig. 3 Differences between events with and without BEC for $x = |p|$, $\delta\Phi$ (upper part) and $\delta\Phi^\star$ (lower part) for models and BEC weights presented in Fig. 1 (with the same meaning of different symbols). Circles denote CAS and stars MaxEnt models with, respectively, constant (for open symbols) and Gaussian (for full ones) weights for BEC. Lines indicate attempts of best power-like fits of type $\delta\Phi \propto \langle N^{(-)} \rangle^\delta$ with δ equal to (going from top) to 2.0, 2.0, 1.5, 0.7 for upper panel and 3.6, 3.6, 3.1, 2.2 for lower panel.

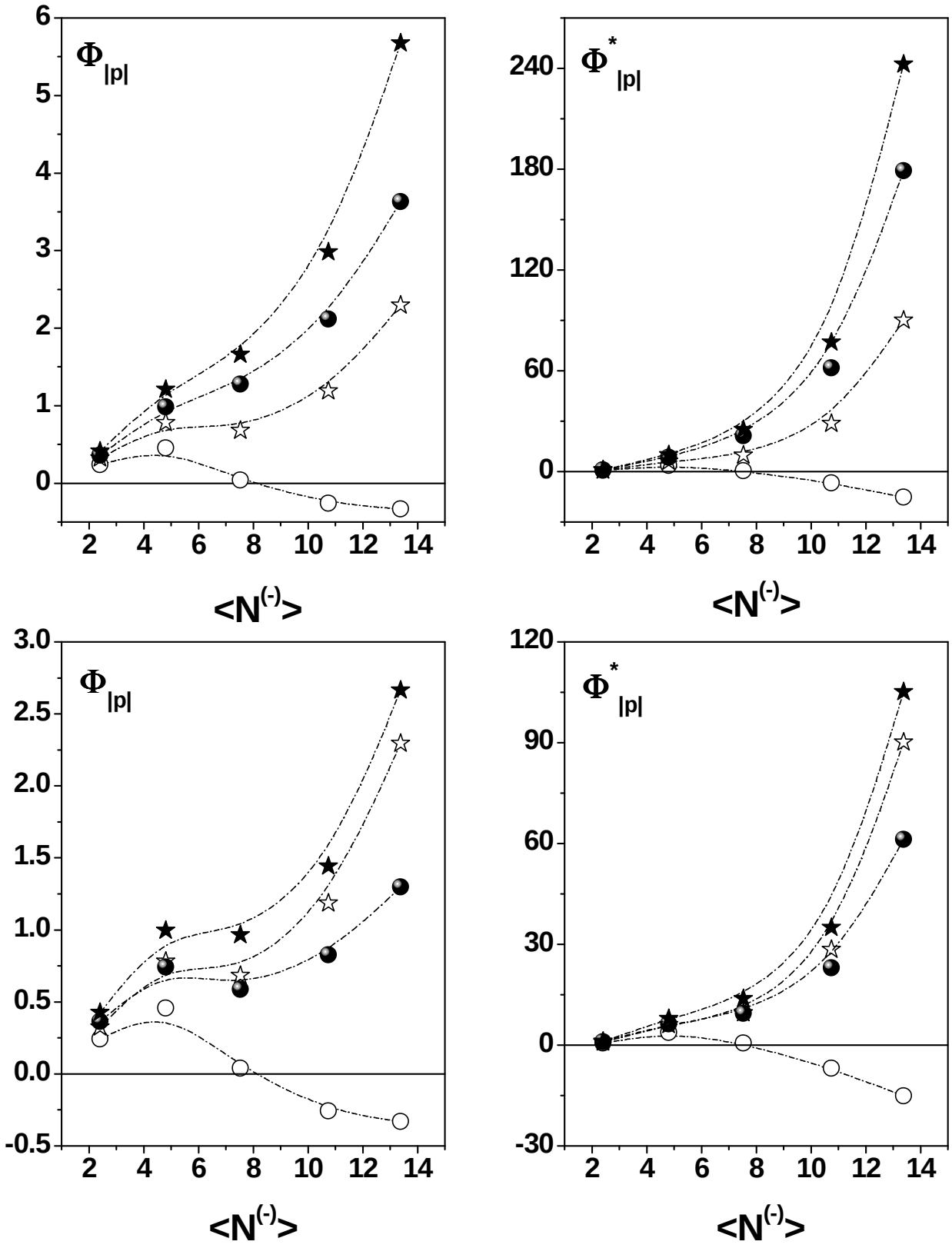


Figure 1:

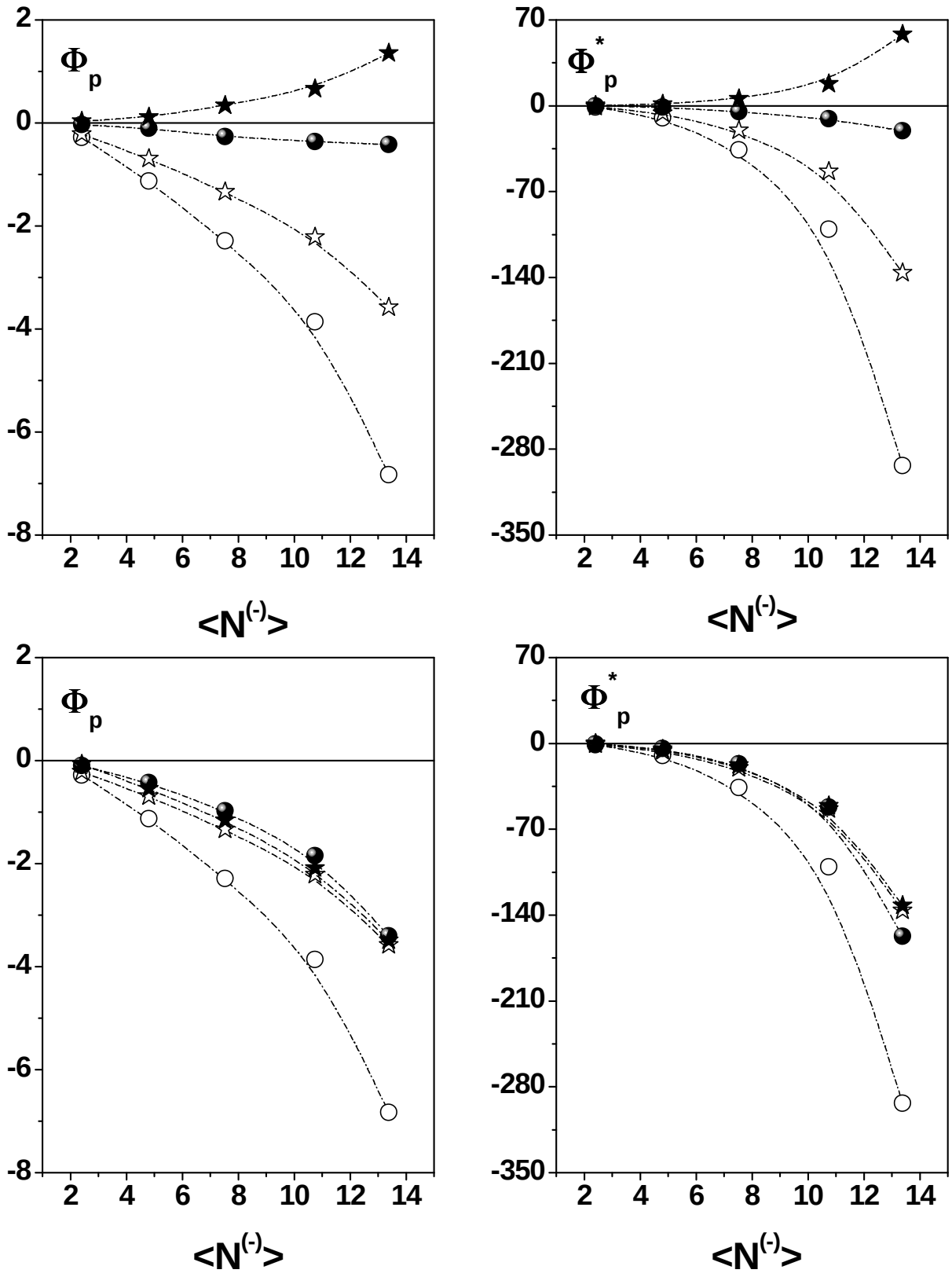
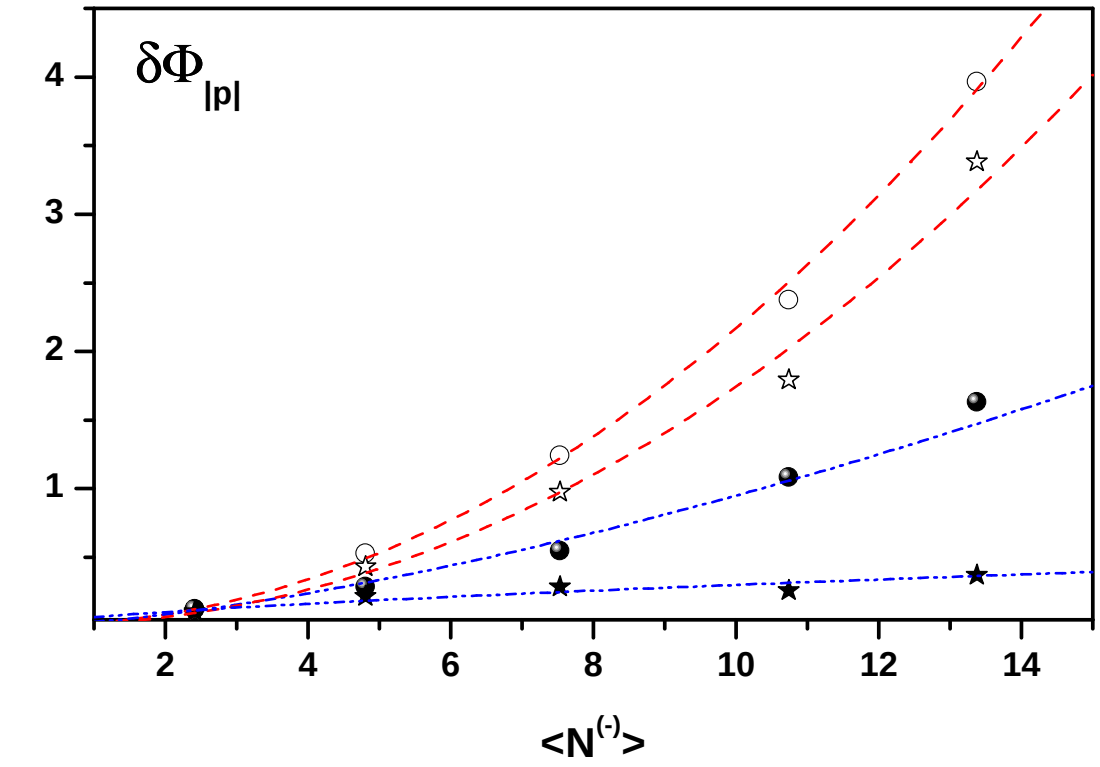


Figure 2:



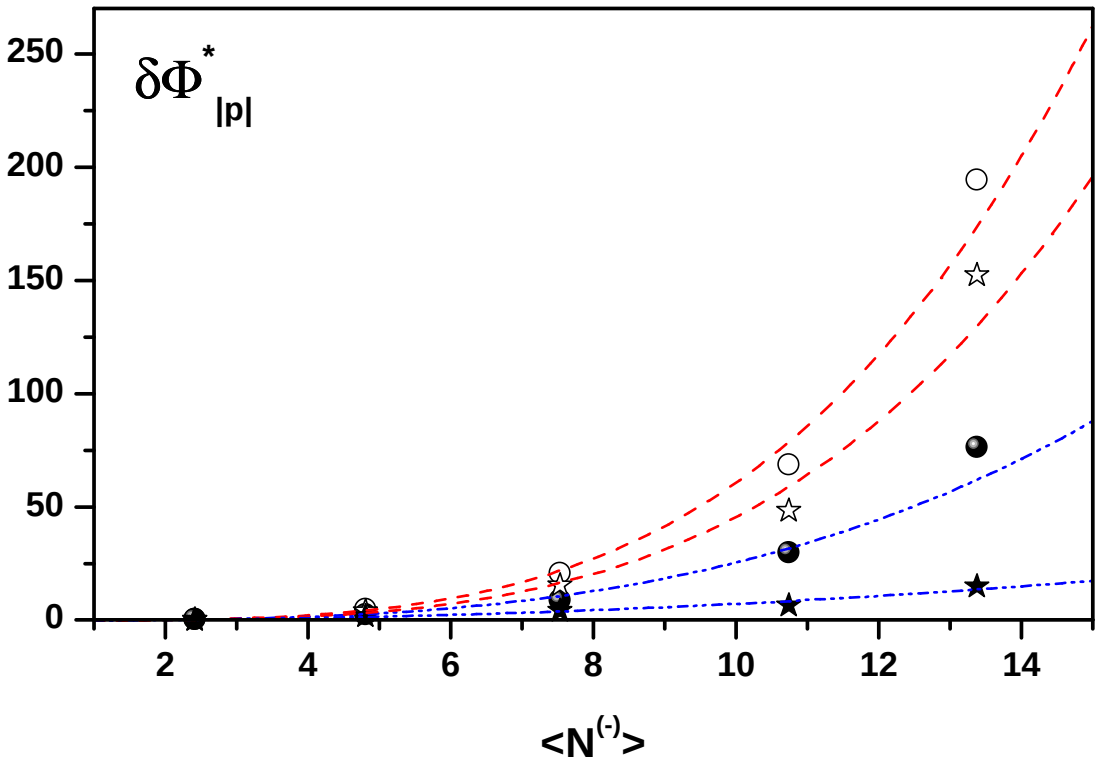


Figure 3: